Eartharea: Surface Area of regions on an ellipsoid Earth

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Contents

Calculating the area of arbitrary regions on the Earth approximated as an ellipsoid. I needed this for conversion between the output of different models.

It's calculated in Emacs Lisp, which showed me that for somewhat complex mathematical tasks Lisp syntax isn't only unproblematic, but actually helps avoiding mistakes. And full unicode support is great for implementing algorithms with ω, λ and ϕ.

1 Intro

For converting between fluxes and emissions I need the area of arbitrary regions made up of longitude×latitude pixels - specifically the transcom regions.

But the earth is no exact sphere, but rather an oblate spheroid. I need to estimate how exact I have to calculate to keep the representation errors of the regions insignificant compared to the uncertainties of the fluxes I work with.

2 Theory

`http://de.wikipedia.org/wiki/Erdfigur` `http://de.wikipedia.org/wiki/Erdellipsoid`

Dadurch nimmt der Meeresspiegel genähert die Form eines Rotationsellipsoids an, dessen Halbachsen (Radien) sich um 21,38 km unterscheiden (a = 6378,139 ± 0,003 km [1], bzw. b = 6356,752 km)

$$f = \frac{a-b}{a} = 1 : 298{,}25642 \pm 0{,}00001 \tag{1}$$

— IERS Conventions (2003).

2.1 Estimating Errors due to spherical approximation

To estimate the errors, just calculate the area of a few samples with different latitude and compare them.

Latitudes

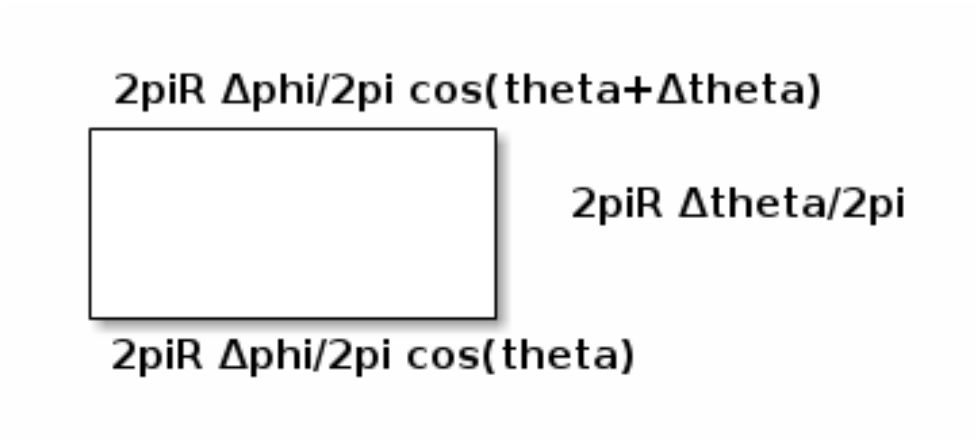
- lat 0°
- lat 10°
- lat 30°
- lat 60°
- lat 85°

Area-Sidelength:

- 0.1°
- 1°
- 4°

2.1.1 Spherical

The simplest case for the latitude-longitude rectangle with latitude theta, longitude phi and earth radius R looks in linear approximation like this:



Using a cylindrical equal area rectangle projection (Lambert) we can calculate the area of a given latitude-longitude square as follows:

$$x = 2\pi R \cdot \Delta\phi \tag{2}$$

$$y = 2\pi R \cdot \Delta\theta \cdot cos\theta \tag{3}$$

$$A = x \cdot y \tag{4}$$

With θ as longitude, ϕ as latitude and R as radius of the earth sphere.

For a 1°×1° square, that equals

$$x = 2\pi R \cdot \frac{1}{360} \tag{5}$$

$$y = 2\pi R \cdot \frac{1}{360} \cdot cos\theta \tag{6}$$

$$A = 4\pi^2 R^2 \cdot \frac{1}{360 \cdot 360} \cdot cos\theta \tag{7}$$

$$A = 4\pi^2 40.589641e^6 km^2 \cdot \frac{1}{129600} \cdot cos\theta; R = 6371km \tag{8}$$

$$A = 4\pi^2 40.589641e^6 km^2 \cdot \frac{1}{129600} \cdot cos\theta; R = 6371km \tag{9}$$

$$A = 12364.3117114km^2 \cdot cos\theta \tag{10}$$

```lisp
(defun spherecutarea (latdeg sidelength deglen)
  "Calculate the area of a cut in a sphere at the latitude LATDEG
with the given SIDELENGTH and the length of one degree at the
Equator DEGLEN."
  (* deglen sidelength ; longitude
     deglen sidelength (cos (* 2 float-pi (/ latdeg 360.0))))) ; latitude

(defun sphere-area (latdeg sidelength)
  "Area of a segment of a sphere at LATDEG with the given
SIDELENGTH."
  (let* ((R 6371.0) ; km
         (deglen (/ (* 2 float-pi R) 360.0))) ; km^2
    (spherecutarea latdeg sidelength deglen)))
```

Listing 1: Calculate the area of a sphere cut

Table 1: Area of lat-lon "square" with the given sidelength as degree in km^2

latitude ↓	0.1	1	4
0	123.64	12364.31	197828.99
10	121.76	12176.47	194823.52
30	107.08	10707.81	171324.93
60	61.82	6182.16	98914.49
85	10.78	1077.62	17241.93

```lisp
(defun spheresegmentarea (latdeg sidelength)
  "Calculate the area of a rectangular segment on a sphere at
latitude LATDEG with the given SIDELENGTH."
  (* 24728.6234228 sidelength sidelength
     (cos (* float-pi (/ latdeg 180.0)))))
```

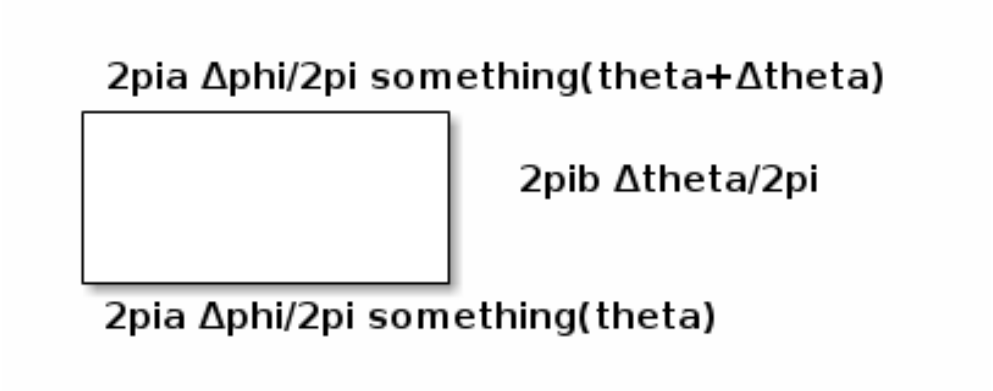
`spheresegmentarea`

2.2 Simple ellipsoid integral (scrapped)

Instead of the very simple spherical compression, we can use integration over the area of an oblated spheroid, or more exactly: an ellipsoid of revolution.

An oblated spheroid has one short axis and two long axi. For the earth, the short axis is the polar radius $b = 6356,752km$ while the long axi have the length of the equatorial radius $a = a = 6378,139 \pm 0,003km$.

Thus the linear approximation of an area on the spheroid looks like this:



Let's scrap that. I'm drowning in not-so-simple ideas, so I'd rather take a pre-generated formula, even if it means cutting leafs with a chainsaw. Let's go to an astronomy book: *Astronomische Algorithmen* by *Jean Meeus* has a formula for distances on an ellipsoid.

2.3 Square approximation with ellipsoid sidelength calculation

Taking the algorithm from *Astronomische Algorithmen rev. 2* by *Jean Meeus*. I want to know how big the errors are when I just take a circle. So let's implement a fitting algorithm.

The following algorithm gives us the distance between two points.

$$F = \frac{\phi_1 + \phi_2}{2}, G = \frac{\phi_1 - \phi_2}{2}, \lambda = \frac{L_1 - L_2}{2} \tag{11}$$

$$S = sin^2 G\ cos^2 G + cos^2 F\ sin^2 \lambda \tag{12}$$

$$C = cos^2 G\ cos^2 G + sin^2 F\ sin^2 \lambda \tag{13}$$

$$tan\omega = \sqrt{\frac{S}{C}} \tag{14}$$

$$R = \frac{\sqrt{SC}}{\omega},\ omega\ in\ radians \tag{15}$$

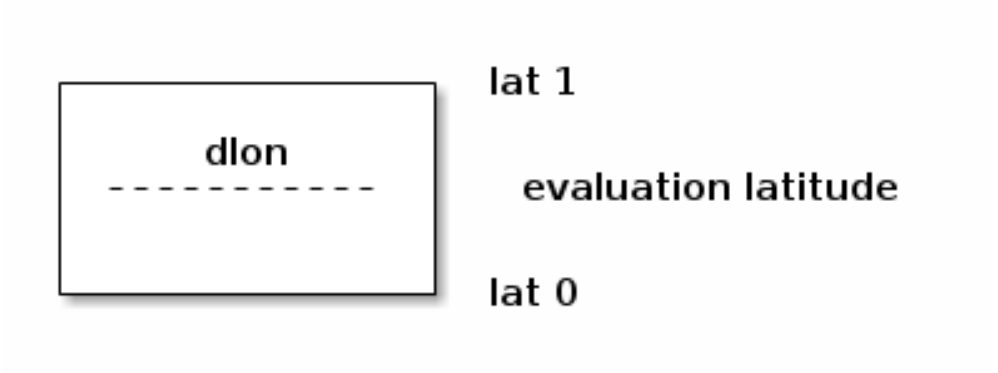
$$D = 2\omega a \tag{16}$$

$$H_1 = \frac{3R-1}{2C}, H_2 = \frac{3R+2}{2S} \tag{17}$$

$$s = D(1 + fH_1\ sin^2F\ cos^2G - fH_2\ cos^2F\ sin^2G) \tag{18}$$

We can now use the distance s between the 4 corners of a pseudo-rectangular area on the ellipsoid to approximate the area of the pseudo-square they delimit.

$$A = \frac{s_{bottomright-bottomleft} + s_{topright-topleft}}{2} \cdot s_{topleft-bottomleft} \tag{19}$$



But by doing so we treat the non-linear problem as linear. To minimize the error, we can split an area into many smaller areas and sum up their areas (numerical approximation).

In following we will use the direct algorithm as well as the numerical approximation.

2.3.1 Code

```
(defmacro turntofloatsingle (var)
  (list 'setq var (list 'float var)))

(defmacro turntofloat (&rest vars)
  "Turn a list of items to floats."
  (cons 'progn (mapcar
                (lambda (var)
                  (list 'turntofloatsingle var))
                vars)))
```

Listing 2: ellipsoid-helpers

```
(defun ellipsoiddistance (a f L1 L2 φ1 φ2)
  "Calculate the distance of two arbitrary points on an ellipsoid.

  Parameters: Equator radius A, oblateness F and for point 1 and
  2 respectively the longitudes L1 and L2 and the latitudes φ1
  and φ2."
  ; ensure that we work on floats
  (turntofloat a f φ1 φ2 L1 L2)
  ; the first simplifications don't depend on each other,
  ; so we just use let to bind them
  (let ((F (/ (+ φ1 φ2) 2))
        (G (/ (- φ1 φ2) 2))
        (λ (/ (- L1 L2) 2)))
    (message (format "F %f G %f λ %f a %f f %f L1 %f L2 %f φ1 %f φ2 %f"
                     F G λ a f L1 L2 φ1 φ2))
    ; the second don't depend on each other either
    (let ((S (+ (* (expt (sin G) 2)
                   (expt (cos λ) 2))
                (* (expt (cos F) 2)
                   (expt (sin λ) 2))))
          (C (+ (* (expt (cos G) 2)
                   (expt (cos λ) 2))
                (* (expt (sin F) 2)
                   (expt (sin λ) 2)))))
      ; now we have a few consecutive definitions, so we use let*
      ; which allows references to previous elements in the same let*.
      (let* ((ω (atan (sqrt (/ S C))))
             (R (/    (sqrt (* S C)) ω)))
        (let ((D (* 2 ω a))
              (H1 (/ (- (* 3 R)) (* 2 C)))
              (H2 (/ (+ (* 3 R)) (* 2 C))))
          ; All prepared. Now we just fit all this together. This is
          ; the last line, so the function returns the value.
          (* D (-
                (+ 1 (* f H1 (expt (sin F) 2) (expt (cos G) 2)))
                (* f H2 (expt (cos F) 2) (expt (sin G) 2)))))))))
```

Listing 3: ellipsoid-distance

```
(defun ellipsoidrectanglearea (a f longitude latitude dlon dlat)
  (let ((L1 longitude)
        (L2 (+ longitude dlon))
        (φ1 latitude)
        (φ2 (+ latitude dlat)))
    (let ((lenlower (ellipsoiddistance a f L1 L2 φ1 φ1))
          (lenupper (ellipsoiddistance a f L1 L2 φ2 φ2))
          (lenwestern (ellipsoiddistance a f L1 L1 φ1 φ2))
          (leneastern (ellipsoiddistance a f L2 L2 φ1 φ2)))
      (if (not (= lenwestern leneastern))
          (error "Western and Eastern length are not equal.
This violates the laws of geometry. We die. Western: %f Eastern: %f"
                 lenwestern leneastern))
      (let ((horizontalmean (/ (+ lenlower lenupper) 2)))
        ; now just return length times width
        (* horizontalmean lenwestern)))))
```

Listing 4: ellipsoid-rectanglearea

```
<<ellipsoid-helpers>>
<<ellipsoid-distance>>
<<ellipsoid-rectanglearea>>

(defun ellipsoidrectangleareafromdeg (latdeg sidelength)
  "Calculate the rectangle area from the latitude LATDEG and the
SIDELENGTH given as degrees."
  (message (format "latdeg %f sidelength %f" latdeg sidelength))
  (let ((londeg 15) ; irrelevant due to symmetry
        (dlondeg sidelength)
        (dlatdeg sidelength)
        (a 6378.139)
        (f (/ 1 298.25642)))
    (let ((lon (* 2 float-pi (/ londeg 360.0))) ; 2π / 360
          (dlon (* 2 float-pi (/ dlondeg 360.0)))
          (lat (* 2 float-pi (/ latdeg 360.0)))
          (dlat (* 2 float-pi (/ dlatdeg 360.0))))
      (ellipsoidrectanglearea a f lon lat dlon dlat))))

(defun ellipsoidrectangleareafromdegnumericalintegration (latdeg sidelength steps)
  "Calculate the rectangle area from the latidute LATDEG and the
  SIDELENGTH given as degrees by adding them in STEPS smaller steps per sidelength."
  (let ((area 0)
        (smallerside (/ (float sidelength)
                        (float steps))))
    (loop for i from 0 to (1- steps) by 1 do
          (message (format "i %f" i))
          (let ((smallerlat (+ latdeg (* smallerside i))))
            ; add stepts times the area since the longitudinal
            ; calculation does not change, so we only need to
            ; calculate in once.
            (setq area (+ area (* steps
                                  (ellipsoidrectangleareafromdeg
                                   smallerlat smallerside))))))

    area))
; no return value
nil
```

Listing 5: Calculate the area of 1x1 degree segments on an ellipsoid

2.3.2 Results for direct calculation

2.3.3 Results for summing up smaller squares.

1. 100 squares per area (10 latitude steps)
2. 10000 squares per area (100 latitude steps)
3. 1000000 squares per area (1000 latitude steps)

Table 2: Area of lat-lon "square" with the given sidelength in km^2, direct

latitude ↓	0.1	1	4
0	123.9203	12391.0741	198026.1548
10	121.9815	12179.9838	193733.6082
20	116.2724	11592.4962	183457.2705
30	106.9937	10649.2503	167557.9088
40	94.4643	9382.6379	146580.1726
45	87.1079	8640.9234	134399.3469
50	79.1019	7834.8158	121219.4843
60	61.4002	6055.4408	92285.3707
70	41.9067	4099.4608	60666.5036
80	21.2036	2025.2137	27301.2374
85	10.5861	962.5255	10264.8590
90	0.1071	107.0494	6844.8700

4. 10 steps vs 1 step, relative

$$\frac{A_{10} - A_1}{A_1} \tag{20}$$

5. 100 steps vs 10 steps, relative

$$\frac{A_{100} - A_{10}}{A_{10}} \tag{21}$$

6. 1000 steps vs 100 steps, relative

$$\frac{A_{1000} - A_{100}}{A_{100}} \tag{22}$$

3 Implementation

This is almost done in the theory. Only thing left to do: Use the algorithm to generate a list of areas per 1° latitude and pass that to a python script which writes it into a netCDF4 file for later usage.

I need a python snippet which takes a list of values from lat 0° to lat 90° as input and turns it into a 360°×180° map.

Or I could just write the data from the elisp code to a file and read that.

Table 3: Area of lat-lon "square" with the given sidelength in km^2 sum 10

latitude ↓	0.1	1	4
0	123.9203	12391.3918	198107.5151
10	121.9815	12180.3007	193814.7025
20	116.2724	11592.8099	183537.3359
30	106.9937	10649.5549	167635.3476
40	94.4643	9382.9239	146652.3820
45	87.1079	8641.1954	134467.6868
50	79.1019	7835.0702	121283.0172
60	61.4002	6055.6486	92336.3892
70	41.9067	4099.6076	60701.4093
80	21.2036	2025.2881	27317.3197
85	10.5862	962.5611	10270.9364
90	0.1071	107.0533	6848.9244

3.1 Write data files

```
<<ellipsoidrectangleareafromdeg>>
(with-temp-file "transcomellipticlat90-sum1000.dat"
  ; switch to the opened file
  (switch-to-buffer (current-buffer))
  (loop for lat from 0 to 90 do
        (insert (concat (number-to-string lat) " "))
        (insert (number-to-string
                 (ellipsoidrectangleareafromdegnumericalintegration lat 1 1000)))
        (insert "\n")))
; dang, this is beautiful!
```

Listing 6: Write integrated ellisoid surface data with 1000 integration steps to a file

```
<<ellipsoidrectangleareafromdeg>>
(with-temp-file "transcomellipticlat90-direct.dat"
  ; switch to the opened file
  (switch-to-buffer (current-buffer))
  (loop for lat from 0 to 90 do
        (insert (concat (number-to-string lat) " "))
        (insert (number-to-string
                 (ellipsoidrectangleareafromdegnumericalintegration lat 1 1)))
        (insert "\n")))
```

Listing 7: Write integrated ellisoid surface data with a single integration step to a file

Table 4: Area of lat-lon "square" with the given sidelength in km^2 sum 100

latitude ↓	0.1	1	4
0	123.9203	12391.3950	198108.3283
10	121.9815	12180.3039	193815.5131
20	116.2724	11592.8130	183538.1364
30	106.9937	10649.5580	167636.1220
40	94.4643	9382.9268	146653.1043
45	87.1079	8641.1981	134468.3705
50	79.1019	7835.0727	121283.6529
60	61.4002	6055.6507	92336.8997
70	41.9067	4099.6090	60701.7587
80	21.2036	2025.2888	27317.4807
85	10.5862	962.5615	10270.9973
90	0.1071	107.0534	6848.9650

```
<<ellipsoidrectangleareafromdeg>>
(with-temp-file "transcomellipticlat90-sum1000vsdirect.dat"
  ; switch to the opened file
  (switch-to-buffer (current-buffer))
  (loop for lat from 0 to 90 do
        (insert (concat (number-to-string lat) " "))
        (insert (number-to-string
                 (- (ellipsoidrectangleareafromdegnumericalintegration lat 1 1000)
                    (ellipsoidrectangleareafromdegnumericalintegration lat 1 1))))
        (insert "\n")))
```

Listing 8: Write the difference between 1000 integration steps and one single step to a file

```
<<ellipsoidrectangleareafromdeg>>
<<spherearea>>
(with-temp-file "transcomellipticlat90-sum1000vssphere.dat"
  ; switch to the opened file
  (switch-to-buffer (current-buffer))
  (loop for lat from 0 to 90 do
        (insert (concat (number-to-string lat) " "))
        (insert (number-to-string
                 (- (ellipsoidrectangleareafromdegnumericalintegration lat 1 1000)
                    (spherearea lat 1))))
        (insert "\n")))
```

Listing 9: Write the difference between ellipsoid integration and a simple sphere to a file

Table 5: Area of lat-lon "square" with the given sidelength in km^2 sum 1000

latitude ↓	0.1	1	4
0	123.9203	12391.3950	198108.3365
10	121.9815	12180.3039	193815.5213
20	116.2724	11592.8130	183538.1444
30	106.9937	10649.5580	167636.1297
40	94.4643	9382.9268	146653.1115
45	87.1079	8641.1982	134468.3773
50	79.1019	7835.0728	121283.6592
60	61.4002	6055.6507	92336.9048
70	41.9067	4099.6090	60701.7621
80	21.2036	2025.2888	27317.4823
85	10.5862	962.5615	10270.9979
90	0.1071	107.0534	6848.9654

```
<<sphereearea>>
(with-temp-file "transcomellipticlat90-sphere.dat"
  ; switch to the opened file
  (switch-to-buffer (current-buffer))
  (loop for lat from 0 to 90 do
        (insert (concat (number-to-string lat) " "))
        (insert (number-to-string (sphereearea lat 1)))
        (insert "\n")))
```

Listing 10: Write the area of segments on a sphere to a file

```
(with-temp-file "transcomellipticlat90-sum1000vssphere.dat"
  ; switch to the opened file
  (switch-to-buffer (current-buffer))
  (loop for lat from 0 to 90 do
        (insert (concat (number-to-string lat) " "))
        (insert (number-to-string (- (ellipsoidrectangleareafromdegnumericalintegration lat 1 1000) (sphere
        (insert "\n")))
```

3.2 Write datafiles to netcdf and plot them

Now just readout that file as csv

3.2.1 First define the plotstyle

The following codeblock can be summoned into other code via

Table 6: Area of lat-lon “square” with the given sidelength in km^2 10 vs. 1

latitude ↓	0.1	1	4
0	0.0000%	0.0026%	0.0411%
10	0.0000%	0.0026%	0.0419%
20	0.0000%	0.0027%	0.0436%
30	0.0000%	0.0029%	0.0462%
40	0.0000%	0.0030%	0.0493%
45	0.0000%	0.0031%	0.0508%
50	0.0000%	0.0032%	0.0524%
60	0.0000%	0.0034%	0.0553%
70	0.0000%	0.0036%	0.0575%
80	0.0000%	0.0037%	0.0589%
85	0.0000%	0.0037%	0.0592%
90	0.0000%	0.0037%	0.0592%

<<addplotstyle>>

```
# add map lines
m.drawcoastlines()
m.drawparallels(np.arange(-90.,120.,30.),
                labels=[False,True,True,False])
m.drawmeridians(np.arange(0.,420.,60.),
                labels=[True,False,False,True])
m.drawmapboundary(fill_color='aqua')
```

Listing 11: plotstyle

Table 7: Area of lat-lon "square" with the given sidelength in km^2 100 vs. 10

latitude ↓	0.1	1	4
0	0.000000%	0.000026%	0.000410%
10	0.000000%	0.000026%	0.000418%
20	0.000000%	0.000027%	0.000436%
30	0.000000%	0.000029%	0.000462%
40	0.000000%	0.000030%	0.000493%
45	0.000000%	0.000031%	0.000508%
50	0.000000%	0.000032%	0.000524%
60	0.000000%	0.000034%	0.000553%
70	0.000000%	0.000036%	0.000576%
80	0.000000%	0.000037%	0.000589%
85	0.000000%	0.000037%	0.000592%
90	0.000000%	0.000037%	0.000593%

Table 8: Area of lat-lon "square" with the given sidelength in km^2 1000 vs 100

latitude ↓	0.1	1	4
0	0.000000%	0.000000%	0.000004%
10	0.000000%	0.000000%	0.000004%
20	0.000000%	0.000000%	0.000004%
30	0.000000%	0.000000%	0.000005%
40	0.000000%	0.000000%	0.000005%
45	0.000000%	0.000000%	0.000005%
50	0.000000%	0.000000%	0.000005%
60	0.000000%	0.000000%	0.000006%
70	0.000000%	0.000000%	0.000006%
80	0.000000%	0.000000%	0.000006%
85	0.000000%	0.000000%	0.000006%
90	0.000000%	0.000000%	0.000006%

3.2.2 Now read datafiles

```
import numpy as np
import pylab as pl
import mpl_toolkits.basemap as bm
import netCDF4 as nc
def singlehemispherelats2map(northernlats):
    """Turn the northern lats (0-90) into a map (180,360)."""
    # duplicate the northernlats
    lats = np.zeros((180, ))
    lats[0:90] = northernlats[:0:-1,1]
    lats[90:] = northernlats[1:,1]
    # and blow them up into a map
    lons = np.ones((360, ))
    lats = np.matrix(lats)
    lons = np.matrix(lons)
    mapscaling = lons.transpose() * lats
    mapscaling = mapscaling.transpose()
    return mapscaling

# first read the file
with open("transcomellipticlat90-sum1000.dat") as f:
    northernlats = np.genfromtxt(f, delimiter=" ")
mapscaling = singlehemispherelats2map(northernlats)
with open("transcomellipticlat90-sum1000vsdirect.dat") as f:
    northernlats = np.genfromtxt(f, delimiter=" ")
mapscalingdiff = singlehemispherelats2map(northernlats)
with open("transcomellipticlat90-direct.dat") as f:
    northernlats = np.genfromtxt(f, delimiter=" ")
mapscalingdirect = singlehemispherelats2map(northernlats)
with open("transcomellipticlat90-sphere.dat") as f:
    northernlats = np.genfromtxt(f, delimiter=" ")
mapscalingsphere = singlehemispherelats2map(northernlats)
with open("transcomellipticlat90-sum1000vssphere.dat") as f:
    northernlats = np.genfromtxt(f, delimiter=" ")
mapscalingdiffsphere = singlehemispherelats2map(northernlats)
```


Listing 12: readcsvareafiles

3.2.3 and plot them

```
# several different plots:
<<plotareamapperpixel>>
<<plotareamapperpixeldirect>>
<<plotareamapperpixelerror>>
<<plotareamapperpixelrelerror>>
<<plotareamapperpixelsphereerror>>
```

Listing 13: plotareamaps

```
# plot it for representation
m = bm.Basemap()
m.imshow(mapscaling)
bar = pl.colorbar()
bar.set_label("area per pixel [$km^2$]")
<<addplotstyle>>
pl.title("Surface Area 1x1 [$km^2$]")
pl.savefig("eartharea_1x1.png")
pl.close()
print """\n\n#+caption:Area when summing 1000x1000 smaller areas
[[./eartharea_1x1.png]]"""
```

Listing 14: Plot a map with the integrated area per pixel

```
m = bm.Basemap()
m.imshow(mapscaling)
bar = pl.colorbar()
bar.set_label("area per pixel [$km^2$]")
# summon map style! :)
<<addplotstyle>>
pl.title("Surface Area 1x1, no numerical integration [$km^2$]")
pl.savefig("earthareadirect_1x1.png")
pl.close()
print "\n\n#+caption:Area when using just one square\n[[./earthareadirect_1x1.png]]"
```

Listing 15: Plot a map with the area per pixel calculated without integration

```
m = bm.Basemap()
m.imshow(mapscalingdiff)
<<addplotstyle>>
bar = pl.colorbar()
bar.set_label("area per pixel [$km^2$]")
pl.title("Surface Area 1x1 difference: sum 1000 vs direct [$km^2$]")
pl.savefig("eartharea1000vs1_1x1.png")  # save as a clean netCDF4 file
pl.close()
print "\n\n#+caption:Difference between summing smaller squares",
print "and just using one square\n[[./eartharea1000vs1_1x1.png]]"
```

Listing 16: Plot a map with the absolute error in the area per pixel from not integrating

```
m = bm.Basemap()
m.imshow(np.log(np.abs(mapscalingdiff/mapscaling)))
<<addplotstyle>>
bar = pl.colorbar()
bar.set_label("relative error per pixel, logarithmic")
pl.title("Surface Area 1x1 diff relative: sum 1000 vs direct")
pl.savefig("eartharea1000vs1rel_1x1.png")  # save as a clean netCDF4 file
pl.close()
print """\n\n#+caption:Relative Area Error by not integrating (logscale)
[[./eartharea1000vs1rel_1x1.png]]"""
```

Listing 17: Plot a map with the relative error in the area per pixel from not integrating

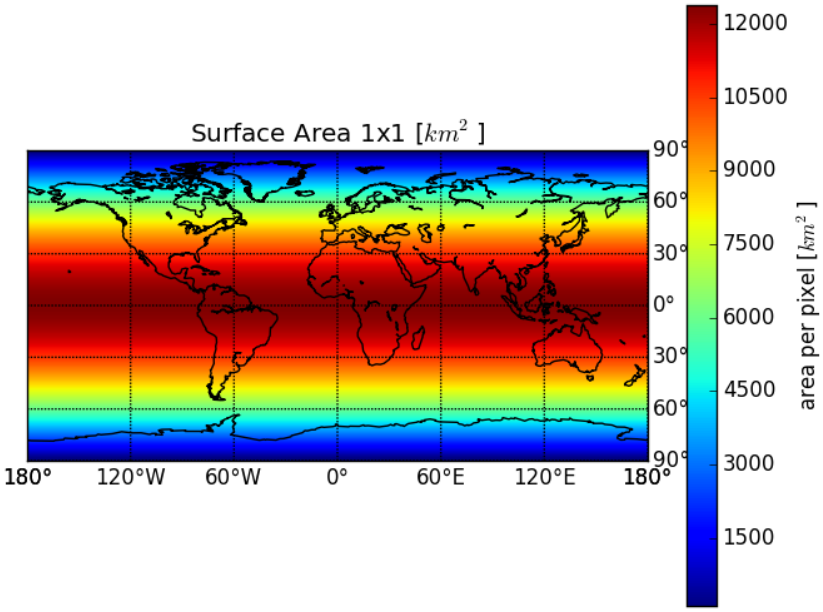
```
m = bm.Basemap()
m.imshow(np.log(np.abs(mapscalingdiffsphere/mapscaling)))
<<addplotstyle>>
bar = pl.colorbar()
bar.set_label("relative error per pixel, logarithmic")
pl.title("Surface Area 1x1 diff relative: sum 1000 vs sphere")
pl.savefig("eartharea1000vsspherere1_1x1.png")
pl.close()
print """\n\n#+caption:Relative Error from Sphere (logscale)
[[./eartharea1000vsspherere1_1x1.png]]"""
```

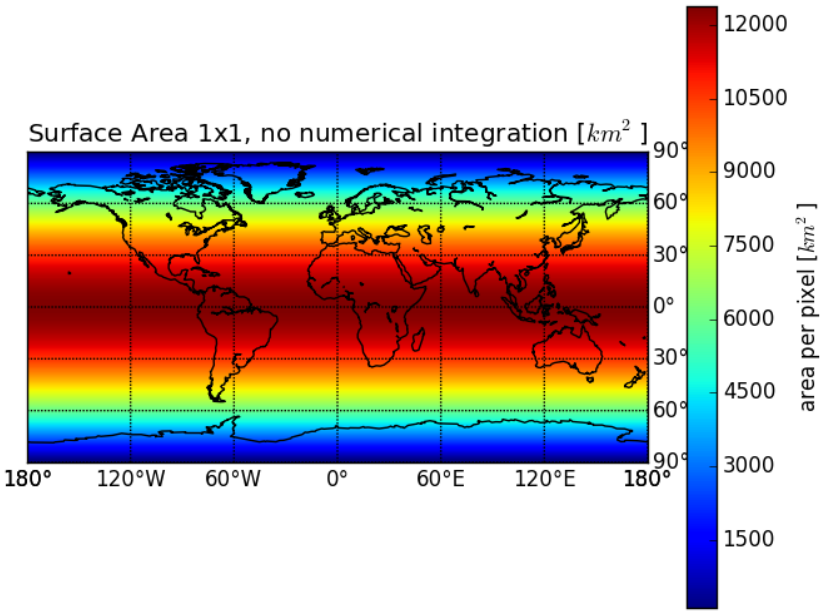
Listing 18: Plot a map with relative error in the the area per pixel from using a spherical approximation

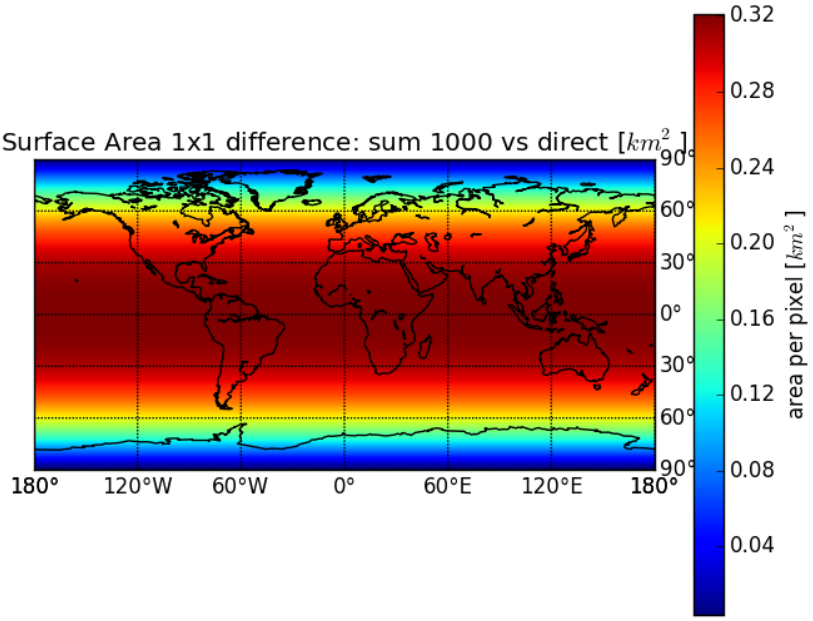
3.2.4 Write the data

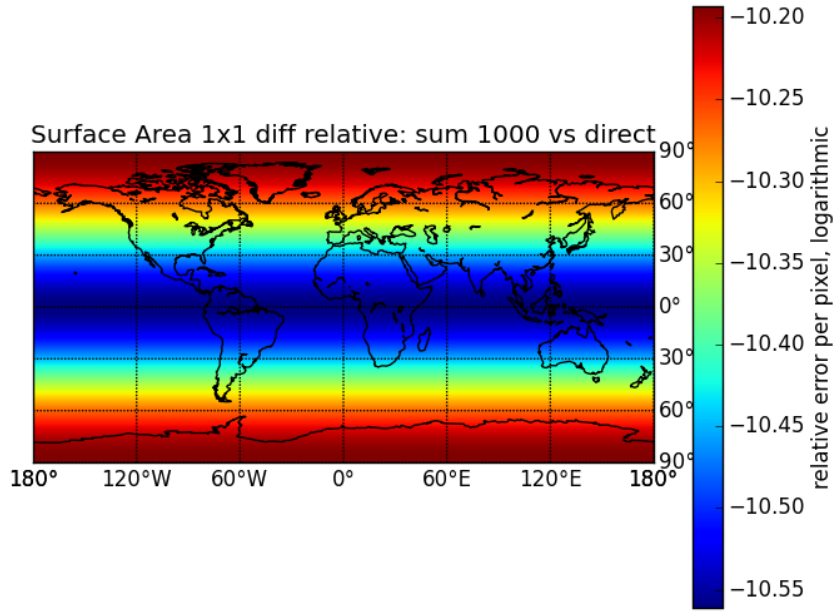
```
<<readcsvareafiles>>
<<plotareamaps>>
D = nc.Dataset("eartharea.nc", "w")
D.comment = "Created with tm5tools/ct2pyshell/transcomareas.org"
D.createDimension("longitude", 360)
D.createDimension("latitude", 180)
area = D.createVariable("1x1", "f8", ("latitude", "longitude"))
area.units = "km^2"
area.comment = "from 180W to 180E and from 90S to 90N"
area[:] = mapscaling
area = D.createVariable("1x1_1000vs1", "f8", ("latitude", "longitude"))
area.units = "km^2"
area.comment = ("Difference between the direct calculation of the "
"area and summing up 1000x1000 smaller areas."
"from 180W to 180E and from 90S to 90N")
area[:] = mapscalingdiff
area = D.createVariable("1x1_direct", "f8", ("latitude", "longitude"))
area.units = "km^2"
area.comment = ("Area calculated without numerical intergration (bigger errors!). "
"from 180W to 180E and from 90S to 90N")
area[:] = mapscalingdirect
area = D.createVariable("1x1_sphere", "f8", ("latitude", "longitude"))
area.units = "km^2"
area.comment = ("Area calculated on a simple sphere. "
"from 180W to 180E and from 90S to 90N")
area[:] = mapscalingsphere
```

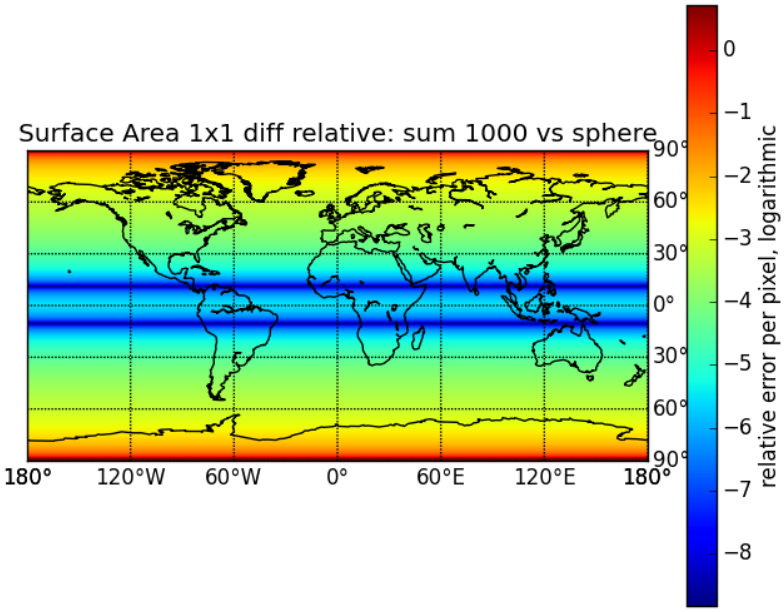
Listing 19: writeearthareanetcdf


Surface Area 1x1 [km^2]
90°
60°
30°
0°
30°
60°
90°
180°
120°W
60°W
0°
60°E
120°E
180°
12000
10500
9000
7500
6000
4500
3000
1500
area per pixel [km^2]


Surface Area 1x1, no numerical integration [km^2]
90°
60°
30°
0°
30°
60°
90°
180°
120°W
60°W
0°
60°E
120°E
180°
12000
10500
9000
7500
6000
4500
3000
1500
area per pixel [km^2]


Surface Area 1x1 difference: sum 1000 vs direct [km^2]
90°
60°
30°
0°
30°
60°
90°
180°
120°W
60°W
0°
60°E
120°E
180°
0.32
0.28
0.24
0.20
0.16
0.12
0.08
0.04
area per pixel [km^2]


Surface Area 1x1 diff relative: sum 1000 vs direct
90°
60°
30°
0°
30°
60°
90°
180°
120°W
60°W
0°
60°E
120°E
180°
−10.20
−10.25
−10.30
−10.35
−10.40
−10.45
−10.50
−10.55
relative error per pixel, logarithmic



4 Validation

4.1 Surface Area of the Earth

Should be around 510 million km²

```
(let ((s 0))
  (loop for lat from 0 to 90 do
        (setq s (+ s (sphere-area lat 1))))
  (/ (* 2 360 s) 1.0e6)) ; million kilometers
```

Listing 20: check-sphere-cuts-sum

```
514.5026761832414
```

```
(let ((s 0))
  (loop for lat from 0 to 90 do
        (setq s (+ s (ellipsoid-rectangle-area-from-degnumerical-integration lat 1 1))))
  (/ (* 2 360 s) 1.0e6)) ; million kilometers
```

Listing 21: check-ellipsoid-cuts

509.55872913305257

```
(let ((s 0))
  (loop for lat from 0 to 90 do
        (setq s (+ s (ellipsoidrectangleareafromdegnumericalintegration lat 1 10))))
  (/ (* 2 360 s) 1.0e6)) ; million kilometers
```

Listing 22: check-ellipsoid-cuts-sum10

509.57373786401286

```
(let ((s 0))
  (loop for lat from 0 to 90 do
        (setq s (+ s (ellipsoidrectangleareafromdegnumericalintegration lat 1 1000))))
  (/ (* 2 360 s) 1.0e6)) ; million kilometers
```

Listing 23: check-ellipsoid-cuts-sum1000

509.5738894527161

4.2 Area of Australia + New Zealand (Transcom Region 10)

Should be around 7,692,024 km^2 + 269,652 km^2 = 7,961,676 km^2

- area of new zealand
- area of australia

```
import netCDF4 as nc
import numpy as np
import pylab as pl

D = nc.Dataset("eartharea.nc")
area = D.variables["1x1"][:]
T = nc.Dataset("../plotting/transcom_regions_ct/regions.nc")
transcom = T.variables["transcom_regions"][:]
mask = transcom[::-1,:] == 10
pl.imshow(mask*area)
bar = pl.colorbar()
bar.set_label("area per pixel [$km^2$]")
pl.title("Area of Australia and New Zealand in [$km^2$] per pixel")
pl.savefig("area-australia.png")
# pl.show()
return np.sum(mask*area)
```

Listing 24: plot the area of australia and new zealand

7976938.58492

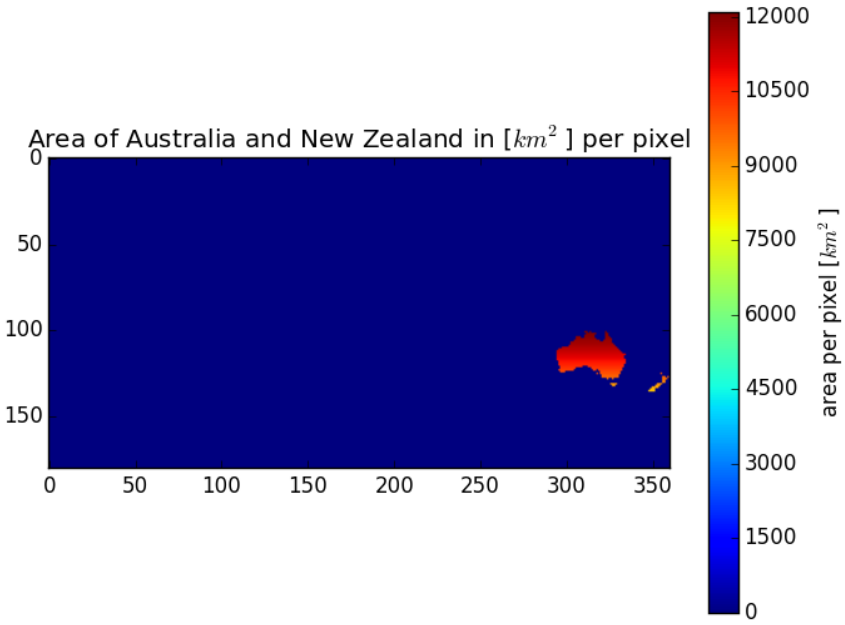


Figure 1: Area of Australia and New Zealand

5 Summary

The area of 1x1 degree pixels on a worldmap in ellipsoid approximation is available in the file eartharea.nc in the variable "1x1". Visualized it looks like this:

To calculate the emissions from mol/m^2, just multiply each gridpoint with 10^6 m^2/km^2 and the gridpoint in the variable:

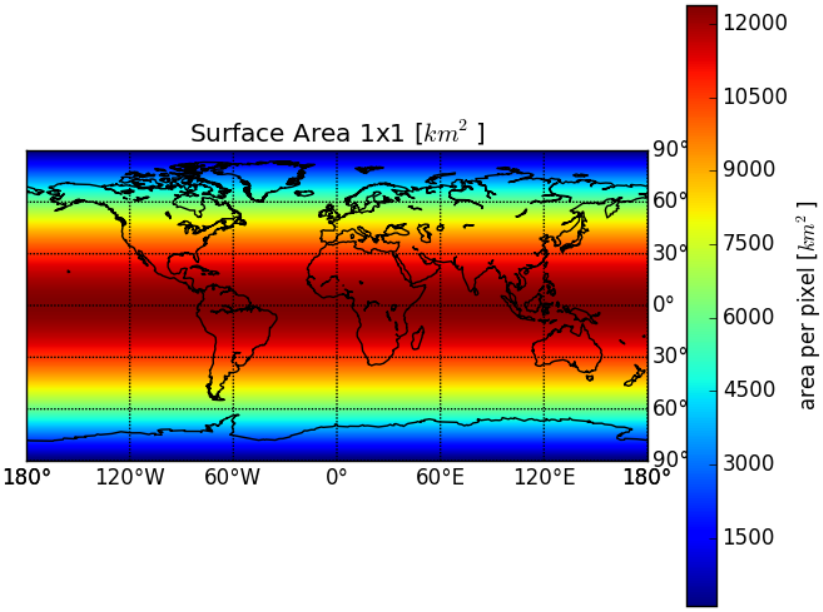


Figure 2: Surface Area of the Earth in km^2

```
<<prep>>
import numpy as np
import pylab as pl
import mpl_toolkits.basemap as bm
import netCDF4 as nc
D = nc.Dataset("eartharea.nc")
area = D.variables["1x1"][:]
flux = np.ones((180, 360)) * np.random.normal(0.0, 1.e-6, (180, 360))
emiss = flux*area
m = bm.Basemap()
m.imshow(emiss)
<<addplotstyle>>
bar = pl.colorbar()
bar.set_label("emissions [mol/s]")
pl.title("random flux $0 \pm 1.e-6 \\frac{mol}{m^{2}s}$ turned to random emissions")
filename = "randomemissions.png"
pl.savefig(filename)
print "#+caption: Random emissions in simple lat/lon plot."
print "[[./" +  filename + "]]"
# plot again, with hobo-dyer projection (equal-area)
pl.close()
m = plotmap(emiss)
<<addplotstyle>>
bar = pl.colorbar()
bar.set_label("emissions [mol/s]")
pl.title("random emissions in hobo-dyer projection")
filename = "randomemissionshobo-dyer.png"

pl.savefig(filename)
print """\n\n#+caption: Random Emissions in Hobo Dyer Projection
[[./""" +  filename + "]]"
```


Listing 25: Plot the Area of the earth with different projections.

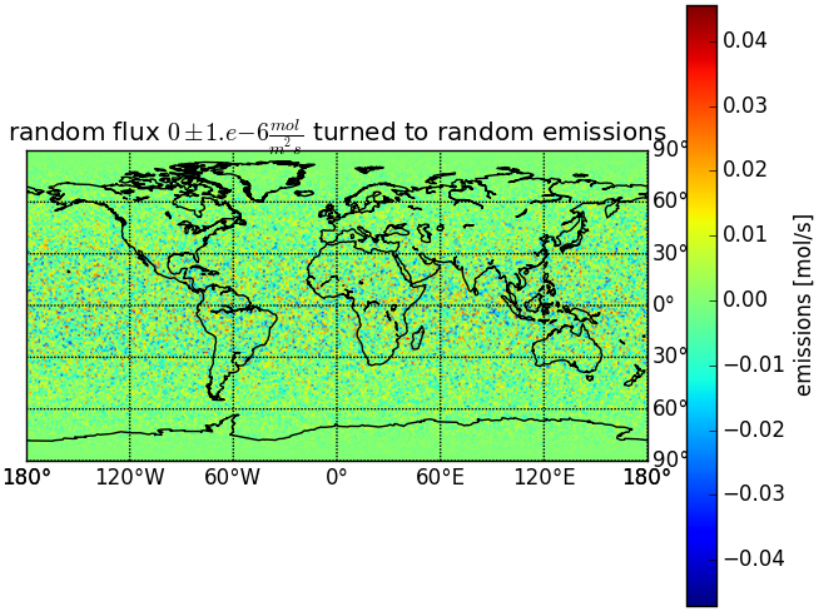


Figure 3: Random emissions in simple lat/lon plot.

```
def plotmap(array):
    """Plot an array as map."""
    m = bm.Basemap(projection='cea', lat_ts=37.5)
    ny, nx = array.shape[:2]
    lons, lats = pl.meshgrid(range(-nx/2, nx/2 + nx%2),
                             range(-ny/2, ny/2 + ny%2))
    x, y = m(lons, lats)
    arr = array.copy()
    for i in arr.shape[2:]:
        arr = arr[:,:,0]
    m.pcolormesh(x, y, arr)
    return m
```

Listing 26: plotmap

5.1 Landarea

Estimating the land area for a given lat-lon region (this requires a land/sea map in the file `t3_regions_landsea.nc`, i.e. from TM5-4DVar, see tm5.sf.net).

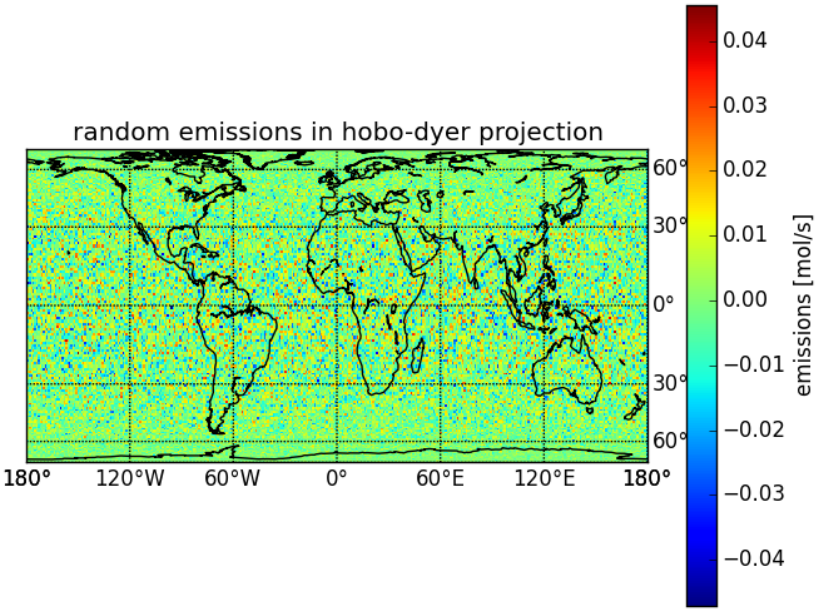


Figure 4: Random Emissions in Hobo Dyer Projection

```
<<prep>>
import netCDF4 as nc
import numpy as np
import pylab as pl
import mpl_toolkits.basemap as bm


def landarea(lat0, lon0, lat1, lon1):
    """Calculate the land area in the rectangle defined by the
    arguments.

    :param lat0: latitude in degree. Southern Hemisphere negative.
    :param lon0: longitude in degree. East negative.

    :returns: landarea within the rectangle in km^2

    >>> samarea = 17.840 * 1000000 # km^2
    >>> ae = landarea(15, -90, -60, -30)
    >>> 0.99 * samarea < ae < 1.01 * samarea
    True
    """
    lat0idx = int(lat0 + 90)
    lat1idx = int(lat1 + 90)
    if lat0idx > lat1idx:
        tmp = lat1idx
        lat1idx = lat0idx
        lat0idx = tmp
    lon0idx = int(lon0 + 180)
    lon1idx = int(lon1 + 180)
    if lon0idx > lon1idx:
        tmp = lon1idx
        lon1idx = lon0idx
        lon0idx = tmp
    D = nc.Dataset("eartharea.nc")
    T = nc.Dataset("t3_regions_landsea.nc")
    area = D.variables["1x1"][:]
    landfraction05x05 = T.variables["LSMASK"][:]
    landfraction1x1 = np.zeros((180,360)) # latxlon
    for i in range(landfraction1x1.shape[0]):
```

6 Notes

6.1 Understanding the macro to turn variables to float

Most of the code snippets here are thanks to ggole in #emacs on irc.freenode.net (What is IRC?).

6.1.1 Single variable

```
(defmacro turntofloatsingle (var)
  (list 'setq var (list 'float var)))
```

Listing 28: Single: Turn a single variable to float

6.1.2 Backtick notation

```
<<turntofloat-single>>
(defmacro turntofloatbackticks (&rest vars)
  "Turn a list of items to floats using backtick notation."
  `(progn ,@(mapcar
             (lambda (var)
               `(turntofloatsingle ,var))
             vars)))
```

Listing 29: Backticks: Turn a list of variables to floats using the backtick notation

6.1.3 Use Mapcar

```
<<turntofloat-single>>
(defmacro turntofloat (&rest vars)
  "Turn a list of items to floats (without using backticks)."
  ; cons turns this into a call of progn on the list returned by
  ; mapcar
  (cons 'progn (mapcar
                (lambda (var)
                  (list 'turntofloatsingle var))
                vars)))
```

Listing 30: mapcar: Turn a list of variables to floats without backtick notation

6.1.4 Common Lisp collect

```
<<turntofloat-single>>
(defmacro turntofloatcollect (&rest vars)
  "Turn a list of items to floats, using the collect directive of loop."
  ; execute progn on the list returned by the loop
  (cons 'progn
        ; loop ... collect returns a list of all the loop results.
        (loop for var in vars collect
              (list 'turntofloatsingle var))))
```

Listing 31: Collect: Turn a list of variables to floats using the Common Lisp collect directive to loop

6.1.5 Explicit List Building

```
<<turntofloat-single>>

; build the list explicitely to make it easier for me to understand
; what the macro does
(defmacro turntofloatexplicit (&rest vars)
  "Turn a list of items to floats (using explicit list building
instead of mapcar)."
  ; prepare an empty list of function calls
  (let ((funclist '()))
    ; for each variable add a call to the single-item macro
    (loop for var in vars do
          ; (list 'turntofloatsingle var) creates the call to
          ; turntofloatsingle with the variable which is referenced by
          ; var. Push puts that at the beginning of the funclist.
          (push (list 'turntofloatsingle var) funclist))
    ; to ensure the right order of operations, we reverse the funclist
    (setq funclist (reverse funclist))
    ; cons turns this into a call of progn on the list. We need progn,
    ; because the funclist contains multiple function calls.
    (cons 'progn funclist)))
```

Listing 32: Explicit List Building: Turn a list of variables to float by building the code list explicitely

6.1.6 Mapcar and Callf

```
<<turntofloat-single>>
; Common Lisp Macro to turn the place to a float in one step.
(defmacro turntofloatinline (&rest places)
  "Turn a list of items to floats using an inline function call."
  `(progn ,@(mapcar
             (lambda (place)
               `(callf float ,place)) places)))
```

Listing 33: Mapcar-callf: Turn a list of variables to floats using mapcar and callf

6.1.7 Test the results

```
<<turntofloat-collect>>
(setq a 1 b 3.8 c 2)
(turntofloatcollect a b c)
(message (number-to-string c))
```

Listing 34: Testing the turntofloat macro results